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Design of M channel Perfect Reconstruction CMFB filter bank for image compression

ANAMIKA JAIN , ADITYA GOEL

Abstract— This paper proposes an optimization method for the design of M channel perfect reconstruction cosine modulated filter (CMFB) bank. The design problem is formulated as a combination of stopband energy, passband energy, coding gain and the least square error of the overall transfer function of the filter bank. The objective function is minimized with the dc leakage free condition, as the dc leakage free condition is essential in image coding application for smooth image. The proposed method is simple, and has high design efficiency in both time and memory requirement. The performance of this algorithm is evaluated in both subjective and objective quality terms. Two design examples are included to illustrate the proposed method.

Index Terms: Coding gain(CG), Cosine modulated filter bank (CMFB), perfect reconstruction (PR), Peak signal to noise ratio (PSNR) ,Pseudo QMF bank, reconstruction error,.

1 INTRODUCTION

Multirate filter banks have been widely used in different image processing fields, such as sub-band coding [1–3], segmentation, object recognition and image and video compression [4-5]. Filter banks design can be considered as design of wavelet bases [6], paraunitary filter bank, orthogonal or biorthogonal filter bank. Previously, two channels QMF bank [6-10] are designed and later on M channel filter banks are designed and implemented for various signal and image processing applications. Although perfect reconstruction is an important parameter in designing filter banks; however, the stopband attenuation can not be made high with such type of constraints thus this condition is relaxed and both PR (perfect reconstruction) and the intra-band aliasing is minimized instead of eliminating fully. [11-13].When this condition is applied for cosine modulated filter bank these systems are called Near Perfect Reconstruction (NPR) CMFB/ pseudo QMF banks. Main advantage of M-channel cosine modulated filter bank is that the analysis and synthesis filters are derived from the prototype filter by cosine-modulation. Designing in this way filter banks become less complex and easy to implement. Many approach such as iterative least square, or standard optimization techniques [11-17], have been already proposed for the design of M-channel CMFB filter banks with perfect and near perfect reconstruction.

In this paper, an approach based on constrained optimization is used to find the coefficients of prototype filter to minimize the combinational objective function.

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The organization of the paper is as follows: in Section 2, a relevant brief analysis of the PR CMFB bank is given. Section 3 describes the formulation of the design problem to obtain the objective function. A brief overview of optimization algorithm is explained in Section 4. In Section 5, two design examples (cases) are presented to illustrate the effectiveness of the proposed method.

2. PROBLEM DESCRIPTION

A typical M-channel maximally decimated filter bank is shown in Figure 1. It splits the input signal *x(n)* into M subband signals, using the low pass, band pass and high pass analysis filters $H_0(z), H_1(z), \ldots H_{M-1}(z)$. These subband signals are down sampled by a factor of M to achieve signal compression or to reduce processing complexity. At the output end, the subband signals are interpolated by a factor of M and passed through synthesis filters, $F_0(z)$, $F_1(z)$, … $F_{M-1}(z)$. The outputs of the synthesis filters are combined to obtain the reconstructed signal x□(n)

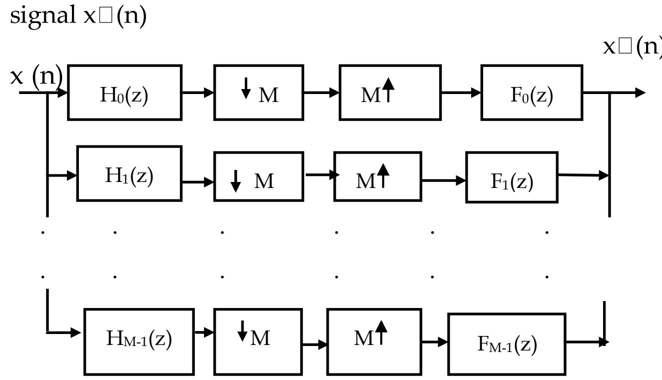


Figure 1: M-channel maximally-decimated filter bank

The reconstructed output of the filter bank can be expressed

$$\hat{X}(z) = \frac{1}{M} T_0(z) X(z) + \frac{1}{M} \sum_{k=0}^{M-1} T_r(z) X(W_M^r) \tag{1}$$

Where $T_0(z)$ is a distortion transfer function, which determines overall amplitude distortion to input signal and $T_r(z)$ is an aliasing distortion transfer function

$$T_r(z) = \frac{1}{M} \sum_{r=0}^{M-1} \sum_{k=0}^{M-1} F_k(z) H(z W_M^r) \tag{2}$$

As shown in [11] when the residual aliasing distortion is neglected, the filter bank transfer function can be given by

$$T(z) = \frac{z^{-N}}{M} \sum_{k=0}^{2M-1} H(z W_{2M}^{k+.5}) H(z^{-1} W_{2M}^{-(k+.5)}) \tag{3}$$

Where N is the order of the prototype filter and $W_{2M}=e^{(-j\pi/M)}$. As in [12] impulse response of analysis and synthesis filter generated from prototype filter h (n) with the aid of cosine modulation to the prototype filter can be expressed as follows

$$h_k(n) = 2h(n)\cos((2k+1)\frac{\pi}{2M}(n - \frac{N-1}{2}) + (-1)^k \frac{\pi}{4}) \tag{4}$$

$$f_k(n) = 2h(n)\cos((2k+1)\frac{\pi}{2M}(n - \frac{N-1}{2}) - (-1)^k \frac{\pi}{4})$$

for k=0,1,…(M-1) and n=0,1,…,N-1.

The length N=2mM of $h_k(n)$ and $f_k(n)$ are the same to satisfy linear phase property [12]. In M-band cosine modulated filter bank,

(i) Perfect reconstruction condition for h[n] can be expressed as

$$\Theta(k) = 2 \sum_{r=0}^{N-2Mk} h(r) h(2Mk + r) \qquad 0 \le k \le E\left\lfloor \frac{N}{2M} \right\rfloor \tag{5}$$

Where $\Theta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$ and $\lfloor E[n] \rfloor$ is the integer part of n.

(ii) for aliasing cancellation stopband attenuation of prototype filter be very high for $\omega_s \ge \frac{\pi}{M}$ as in [12].

(iii) Coding gain of filter banks measures its energy concentration capability. By modeling a natural image as a one-dimensional Markovian source with a correlation factor ρ=.95 and by assuming uncorrelated quantization errors, Katto and Yasuda [20] derived a filter dependent coding gain given by:

$$CG(\rho) = 10\log_{10}(\prod_{k=0}^{M-1}(A_k B_k)^{\alpha_k}) \tag{6}$$

Where: $A_k = \sum_i \sum_i h_k(i)\, h_k(j)\, \rho^{|j-i|}$, $B_k = \sum_i g_k(i)^2$ and h_k and g_k are the k[th] analysis and synthesis filter of the M channel uniform filter bank, a_k is the corresponding subsampling ratio, and ρ is the correlation factor.

(iv) In image coding, dc leakage free condition needed to minimize the checkerboard and ringing artifacts, dc leakage in subbands other than the lowest one is desired. DC-leakage free property in the cosine modulated filter bank can be expressed as

$$h^t c(\omega) = 0, \text{ for } \forall \omega = \frac{(2k+1)\pi}{2M}, 1 \le k \le M-1 \tag{7}$$

Thus M-channel PR cosine modulated filter banks for image compression application are designed by solving the following minimization problem ϕ

$$\varphi = (\alpha E_s + (1-\alpha) E_p) + \beta E_r + \gamma E_{cg} \tag{8}$$

α, β and γ are the parameters to control the characteristics of CMFB filter bank.

3. PROBLEM FORMULATION

For odd N, as in [12], the frequency response of the linear phase prototype filter can be expressed

$$H(e^{j\omega}) = e^{-j\omega\frac{N}{2}} \sum_{n=0}^{\frac{N-1}{2}} h(\frac{N+1}{2} + n) 2\cos(\omega(n+.5)) \tag{9}$$

The amplitude of the frequency response, on a uniform frequency grid of range [ω_s, π], in a matrix form is Cp where

$$p = [h_i(\frac{N+1}{2}), \ldots, h_i(N)]^t \quad C_{i,j} = 2\cos(\omega_{i-1}((j-1)+.5)) \tag{10}$$

for i=1,..L and j=1,…,(N-1)/2.

Stopband energy and Passband energy on Ω equals to

$$E_s = \frac{1}{\pi - \omega_s} \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega \tag{11}$$

$$E_s \approx \frac{1}{L} \sum_n \left|H(e^{j\omega_n})\right|^2 = \frac{1}{L} p^t C^t C p \tag{12}$$

Reconstruction error, aliasing error on Ω equals to

Similarly, reconstruction error and aliasing error

$$E_r = \frac{1}{\pi} \int_0^{\pi} \left|1 - H(e^{j\omega})\right|^2 d\omega \tag{13}$$

$$E_r = \sum_{k=0}^{k=E\left\lfloor \frac{N}{2M} \right\rfloor} \left(\Theta(k) - 2 \sum_{r=0}^{N-2Mk} h(r) h(2Mk + r) \right) \tag{14}$$

Similarly, coding gain for image coding application given as eqn. (6).

Thus, the objective function leads to following expression

$$\phi = \alpha E_s + (1-\alpha) E_p + \beta E_r + \gamma E_{cg} \tag{15}$$

With dc leakage free constraint as given in eqn. (7).

4. THE DESIGN ALGORITHM

In the algorithm presented here, the unit energy constraint [10] is imposed within some pre specified limit. The design algorithm proceeds through the following steps:
(1) Assume initial values of α, β, γ and ω_s
(2) Start with an initial vector $h_0 = [h(0),...,h(N)]^t$ satisfying the unit energy constraint.
(3) Set the function tolerance, convergence tolerance.
(4)Optimize objective function eqn. (15) using constrained optimization method for the specified tolerance.
(5) Evaluate all the analysis and synthesis component filters of cosine modulated filter bank using prototype filter h.
(6) Evaluate all performance parameters such as coding gain, reconstruction error, and stopband attenuation for prototype and component filters of the overall filter bank.

5. RESULTS AND DISCUSSION

M-channel NPR CMFB bank has been designed with the proposed technique using MATLAB. Two design examples are presented to illustrate the effectiveness of the proposed method. The method solved the problem using constrained optimization, starts with initial value zero of the filter coefficients $h_0(n)$ for all n, except that h(N/2–1) = h(N/2) = $2^{-1/2}$, $0 \le n \le N-1$. Only half of the coefficients are to be optimized due to linear phase condition imposed. This choice of the initial value of the filter coefficients satisfies the unit energy constraint and increase the convergence speed. The frequency response characteristics of the prototype filter indicates that the stopband attenuation is increased without sacrificing the flatness of passband response as the filter length is increased. As compared to the Mitra and others [23-24] optimizing the cutoff frequency results in very high order filters having equiripple behaviour in both the passband and stopband. In our method ripples in both passband and the stopband can be controlled independently. Intraband aliasing is minimized by condition of high stopband attenuation for the frequency greater than pi/M. Further, for image coding application shorter filters are required as the high coding gain with least distortion is the main requirement.

The performance analysis of the designing cosine modulated filter bank with the proposed method is tabulated in Table 1.Results show that as the length of the filter increased stopband attenuation and coding gain increased and reconstruction error and aliasing distortion is reduced. As shown in table increase in filter taps increased stopband attenuation but coding gain increased by 0.01 db only when filter taps are increased from 16 to 24. Increasing filter length and introducing more number of channels increase complexity of the system. Thus the 8x16 PR CMFB is used for image coding application. Frequency response of the cosine modulated filter bank for 4 channel and 8 channel cosine modulated filter banks are as shown in figures 2 (a) and (b). Both the figures show that the stopband attenuation with 4x16 CMFB is 25dB and 8x16 CMFB is 23 dB. Further, as the number of taps increased, filter banks result in sharper transition bandwidth which is not the desired design parameter in image compression. Thus the filter bank with less distortion (magnitude and aliasing) is considered for comparison.
The performance of the proposed design technique is evaluated in terms of the following significant parameters:

Amplitude distortion (e_{am})

$$e_{am} = \max_{\omega}(1 - |T_0(e^{j\omega})|) \tag{16}$$

Stop-band edge attenuation

$$A_s = -20\log_{10}(H_0(\omega_s)) \tag{17}$$

A common measure of encoded image quality is the peak signal-to-noise ratio, which is given as

$$PSNR = 20\log_{10}\left(255/\sqrt{MSE}\right) \tag{18}$$

where *MSE* denotes the mean-squared-error.

In all examples, stop-band first lobe attenuation (A_s) has been obtained from the respective zoomed attenuation curves and the constants $\alpha, \beta, \gamma, \varepsilon_1$ have been selected by trial and error method to obtain the best possible results.

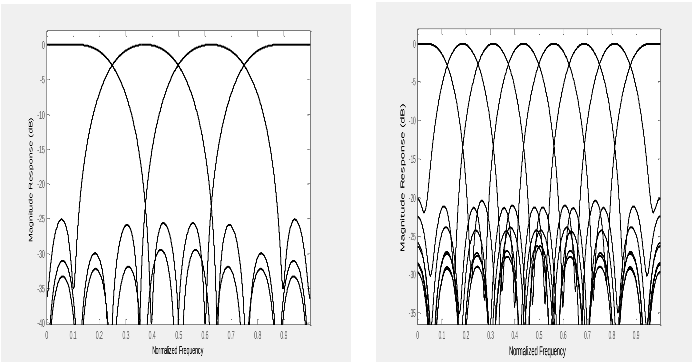
The simulation results presented in the following examples have large stopband attenuation, and very small reconstruction error which can be practically considered as perfect reconstructions. Further, as the length of prototype filter increased, stopband attenuation increases with increased reconstruction error resulting in more aliasing distortion.

5.1. *Case 1*

The prototype filter of 4 band cosine modulated filter banks with subband filter length of 16 was designed using the design method discussed in Section IV. The magnitude response of the prototype filter, the analysis subband filters as defined in eqn. (4) are shown in Fig. 2(a). Large stopband attenuation, approximately equals to 25 dB down, are observed for the subband filters. The reconstruction error is very small with a peak reconstruction error of less than 1.5×10^{-12} dB and aliasing distortion is 300dB.

5.2. *Case 2*

The prototype filter of 8 channel cosine modulated filter banks with subband filter length of 16 was designed using the design method discussed in Section IV. The magnitude response of the prototype filter, the analysis subband filters as defined in eqn. (4), are shown in Fig. 2(b). Large stopband attenuation 23 dB down is observed for the subband filters. The reconstruction error is very small 1.5×10^{-14} dB and aliasing distortion is also small 300dB.



(a) (b)

Fig.2 (a) Frequency Response Of 4x16 (b) 8x16 PR CMFB Filter Bank

5.4 IMAGE CODING RESULTS

We test our designed PR CMFB filter bank with uniform quantizer image coder on the standard 512x512 Lena and Barbara image. We compare the results with dct based image coder

Table I
Performance analysis of the PR CMFB filter bank

M	N	Es	Er	Coding Gain
4	16	25 dB	1.5×10^{-12}dB	8.20
8	16	23dB	1.5×10^{-14} dB	9.35
8	24	26.026 dB	2.0×10^{-14} dB	9.36

Table 2
Performance analysis of the PR CMFB filter bank with Uniform quantizer for 'Lena' image

Rate	PSNR(dB)	
(bpp)	DCT(8X8)	proposed PR CMFB (Uniform Quantizer)
0.25	34.02 dB	34.42dB
0.5	37.41 dB	37.60 dB
0.75	39.54 dB	39.62 dB
1.00	41.09 dB	41.14 dB

and Standard JPEG coder using dct. Coding results for Lena image at different rates are listed in Table2. From Table2 we see that our proposed filter bank consistently outperforms the baseline JPEG by a large margin approx.2.0dB. It is also 0.05-0.4dB better than the improved dct based uniform quantizer. Although our proposed coder falls short when compared to the wavelet based coder for Lena at 1b/p, still it performs better. Visual results for Lena and Barbara image are shown in Fig 3(a), (b), (c) and (d). Visual quality of both images indicates the superiority of the proposed filter bank. Our proposed dct based image coder is very competitive in terms of quality, simplicity and computational complexity.

(a) (b) (c) (d)

Fig.3.(a) Original 'Barbara'image (b) reconstructed 'Barbara' image (c) Original 'Lena' image (b) reconstructed 'Lena' image at 0.31 bpp.

6. CONCLUSION

This paper presents a method to design perfect reconstruction cosine modulated filter bank for image compression. The design problem is formulated as an optimization problem as in [13]. However, due to inclusion of the dc leakage free constraint both stopband attenuation and coding gain are not much higher, still visual quality of the images improves. The proposed method is applied to design prototype filters for cosine modulated filter banks giving highest priority to coding gain. Design examples are presented for low order cosine modulated filter banks design using the proposed algorithms. High performance filter banks with small reconstruction error and large coding gain with moderate stopband attenuation were obtained. Although performance of the designed filter bank outperforms dct based image coder at low bit rates however, is not better than wavelet based image coder. Noted the presented algorithm can be used to design biorthogonal cosine modulated filter banks, as well

REFERENCES

[1] D. Esteban, C. Galand, "Application of quadrature mirror filter to split band voice coding schemes", in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ASSP)*, 1977, pp. 191–195.

[2] M.J.T. Smith, S.L. Eddins, "Analysis/synthesis techniques for sub-band image coding", *IEEE Trans. Acoust. Speech Signal Process. ASSP*-38 (8) (1990) 1446–1456.

[3] J.W. Woods, S.D. O'Neil, "Sub-band coding of images", *IEEE Trans. Acoust. Speech Signal Process. ASSP-34* (10) (1986) 1278–1288.

[4] T.D. Tran, T.Q. Nguyen, "On *M*-channel linear phase FIR filter banks and applications in image compression", *IEEE Trans. Signal Process.*45 (9) (1997) 2175–2187.

[5] T. Xia, Q. Jiang, "Optimal multifilter banks: Design related symmetric extension transform and application to image compression", *IEEE Trans.Signal Process.* 47 (7) (1995) 1878–1889.

[6] S.C. Chan, C.K.S. Pun, K.L. Ho, "New design and realization techniques for a class of perfect reconstruction two-channel FIR filter banks and wavelet bases", *IEEE Trans. Signal Process.* 52 (7) (2004) 2135–2141.

[7] H. Xu, W.S. Lu, A. Antoniou, "An improved method for the design of FIR quadrature mirror image filter banks", *IEEE Trans. Signal Process.*46 (5) (1998) 1275–1281.

[8] R. Bregovic, T. Saramaki, "A general purpose optimization approach for designing two-channel FIR filter banks", *IEEE Trans. Signal Process.*51 (7) (2003) 1783–1791.

[9] C. K. Chen and J. H. Lee, "Design of quadrature mirror filters with linear phase in the frequency domain," *IEEE Trans. Circuits Syst. II*, vol. 39,pp. 593–605, Sept. 1992.

[10] V.K. Jain, R.E. Crochiere, "Quadrature mirror filter design in time domain", IEEE Trans. Acoust. Speech Signal Process. ASSP-32 (4) (1984) 353–361.

[11] T. Saramaki and R. Bregovic, "An efficient approach for designing nearly perfect reconstruction cosine-modulated and modified DFT filter banks," in *Proc. IEEE ICASSP*, vol. 6, 2001, pp. 3617–3620.

[12] M. Rossi, J. Y. Zhang, and W. Steenarrt, "Iterative constrained least squares design of near perfect reconstruction pseudo QMF banks," in *Proc. CCECE'96*, 1996, pp. 766–769.

[13] M. Rossi, J. Y. Zhang, and W. Steenarrt, "Iterative constrained least squares design of perfect reconstruction QMF banks," in *Proc. CCECE'96*, 1996, pp. 766–769.

[14] T. Q. Nguyen, "Digital filter bank design quadratic constrained formulation,"*IEEE Trans. Signal Processing*, vol. 43, pp. 2103–2108, Sept.1995.

[15] H. Xu,W. S. Lu, and A. Antoniou, "Efficient iterative design method for cosine-modulated QMF banks," *IEEE Trans. Signal Processing*, vol. 44, pp. 1657–1668, July 1996.

[16] C. K. Goh and Y. C. Lim, "An efficient algorithm for the design of weighted minimax M-channel cosine-modulated filter banks," *IEEE Trans. Signal Processing*, vol. 46, pp. 1426–1430, May 1998.

[17] C. S. Burrus, J. A. Barreto, and I. W. Selesnick, "Iterative reweighted least squares design of FIR filters," *IEEE Trans. Signal Processing*, vol.42, pp. 2926–2936, 1994.

[18] E. Abdel-Raheem, F. El-Guibaly, and A. Antoniou, "Efficient design of cosine-modulated filter banks," in *Proc. IEEE Pacific Rim Conf.on Communications, Computers, and Signal Processing*, 1995, pp. 387–390.

[19] R. D. Koilpillai and P. P. Vaidyanathan, "Cosine-modulated FIR filter banks satisfying perfect reconstruction," *IEEE Trans. Signal Processing*,vol. 40, pp. 770–783, Apr. 1992.

[20] J. Katto and Y. Yasuda, "Performance evaluation of subband coding and optimization of its filter coefficients," in *Proc. SPIE Visual Communication and Image Processing*, Boston, MA, Nov. 1991, pp.95–106.

[20] T. Nagai, C.W. Kok, M. Ikehara, and T. Q. Nguyen, "Design and lattice structure of FIR paraunitary filter banks with linear phase," *IEICE Trans.Fund. Electron. Commun. Comput. Sci.*, pp. 712–721, Apr. 1997.

[21] P.P. Vaidyanathan, Multirate Systems and Filter Banks, Prentice Hall, Englewood Cliffs, NJ, 1993.

[22] Charles D. Creusere and Sanjit K. Mitra, "A simple method for designing high-quality prototype filters for *M* band pseudo QMF banks", *IEEE Trans. Signal Processing*, **43**, 1005–1007 (1995).

[23] Yuan-Pei Lin and P. P. Vaidyanathan, "A Kaiser window approach for the design of prototype filters of cosine modulated filterbanks", *IEEE Signal Processing Lett.*, **5**, 132–134 (1998).

[24] J. D. Johnston, "A filter family designed for use in quadrature mirror filter banks", *Proc. IEEE Int. Conf.Acoustics, Speech, Signal Processing*, Denver, 291–294 (1980).

[25] A.K. Jain, Fundamentals of Digital Image Processing. Englewood Cliffs, NJ: Prentice-Hall, 1989.